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FREE CONVECTION HEAT TRANSFER THROUGH COUPLED THERMAL BOUNDARY LAYERS IN A PARTITIONED TRIANGULAR ENCLOSURE

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ABSTRACT

A numerical investigation has been carried out for the coupled thermal boundary layers on both sides of a partition placed in an isosceles triangular enclosure along its middle symmetric line. The working fluid is considered as air which is initially quiescent. A sudden temperature difference between two zones of the enclosure has been imposed to trigger the natural convection. It is anticipated from the numerical simulations that the coupled thermal boundary layers development adjacent to the partition undergoes three distinct stages; namely an initial stage, a transitional stage and a steady state stage. Time dependent features of the coupled thermal boundary layers as well as the overall natural convection flow in the partitioned enclosure have been discussed and compared with the non-partitioned enclosure. Moreover, heat transfer as a form of local and overall average Nusselt number through the coupled thermal boundary layers and the inclined walls is also examined. The details results will be discussed in the full paper.

Keywords: Partition, Natural Convection, Boundary Layer, Triangular Enclosure, Nusselt Number.

1. INTRODUCTION

Natural convection in enclosures is a topic of considerable interest for the engineers. This is applicable to many situations in nature and engineering, e.g. thermal design of buildings, to cryogenic storage, solar collector design, nuclear reactor design, and others. There exists several excellent reviews [l , 2] in the literature. Most of the studies devoted to an enclosure with no partitions. However, there have been several studies on the effect of vertical partition in the rectangular enclosure in suppressing natural convection, including the case of a porous medium [3-10]. This problem is of fundamental importance for a variety of reasons. For engineers who deal with insulation in the building, it is important for them to know the net heat transfer rate across solid walls and windows separating a warm room from a colder environment or vice versa. Moreover, from the point of view of fundamental research in heat transfer and fluid mechanics, it is also important to understand the interaction of two convective systems coupled across a partially conducting wall. Previous studies have shown that for a laminar flow regime even though the partition is perfectly conducting, it depresses natural convection in the cavity in comparison with that in a non-partitioned cavity and thus heat transfer through the cavity is significantly reduced ([11–14]).

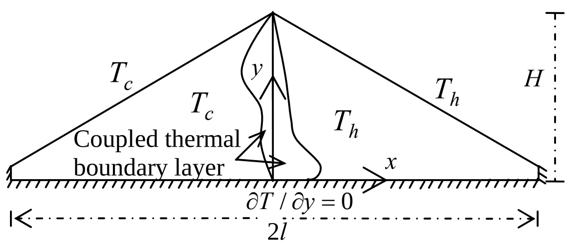
Some other studies which considered the natural convection in rectangular enclosures with multiple vertical partitions [14-16] are also available in the literature. In addition to the rectangular or square cavity, the attention has also been given to the triangular enclosure [17-21] recently for its various engineering applications. The studies related to attic space have been mostly devoted to two types of boundary conditions; day time or summer time condition [18, 20], where the top inclined surfaces are heated and the horizontal bottom surface is cooled or adiabatic and night time or winter time condition [17, 21], where the upper inclined surfaces are cooled and the bottom surface is heated or adiabatic.

The issue of heat transfer through the top surfaces of the attic spaces is the main concern of building a house to give thermal comfort to the occupants. To the best of the authors' knowledge there exists no study on partition triangular cavities to suppress of heat transfer through the inclined walls. The lack of such investigation has motivated the current study to compare the heat transfer phenomena for partitioned and non-partitioned triangular enclosures. The obtained results could help the builders for their insulation system in the attic shaped houses. The emphasis has been given to the transient process of natural convection resulting from a suddenly generated temperature difference between the fluids on the two sides of a conducting partition which has been placed along the geometric centre line of the enclosure. The impact of the partition on transient heat transfer is also discussed in this study.

2. FORMULATION OF THE PROBLEM

Under consideration is a triangular cavity of height H, half length of the base l, containing a Newtonian fluid, air which is initially at quiescent. A partition is placed along the geometric centre line of the enclosure. Two interiors of both sides of the partition together with adjacent inclined walls receive different temperature with the left side receiving cold and the right side receiving hot temperature after time t = 0. In order to avoid the singularities at the tips in the numerical simulation, the tips are cut off by 5% and at the cutting points (refer to Fig. 1) rigid non-slip and adiabatic vertical walls are assumed. The bottom surface is also considered as adiabatic and rigid non-slip. We anticipate that this modification of the geometry will not alter the overall flow development significantly. For the present case H = 0.14 m and the l = 0.28 m. The origin of the coordinate is where the partition and the bottom surface intersect.

The development of natural convection inside an attic space is governed by the following two-dimensional Navier–Stokes and energy equations with the Boussinesq approximation:



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta(T-T_0) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where u and v are the velocity components along x- and y-directions, t is the time, p is the presure, ν, ρ, β and κ are kinematic viscosity, density of the fluid, coefficient of thermal expansion and thermal diffusivity respectively, g is the acceleration due to gravity and T is the temperature.

Here, the three dimensionless parameters which govern the flow are the Rayleigh number, Ra, the Prandtl number, Pr and the aspect ratio, A defined respectively as

$$Ra=\frac{g\beta\Delta T H^3}{\kappa\nu} \tag{5}$$

$$\Pr=\frac{\nu}{\kappa} \tag{6}$$

$$A=\frac{H}{l} \tag{7}$$

Note that in this study the working fluid is air with a fixed Prandtl number of Pr = 0.72; the aspect ratio of the enclosure is A = 0.5; and the Rayleigh number also is fixed at 10^8. This Rayleigh number is calculated based on the temperature difference for the entire enclosure ($\Delta T = T_h$-T_c). As shown in Fig. 1, the bottom surface of the enclosure is adiabatic; the two inclined surfaces are isothermal and fixed at T_c and T_h respectively; the partition of a zero thickness is vertically placed in the geometric centreline of the enclosure and is diathermal for which only horizontal heat transfer is considered (refer to [6,15]) and all three surfaces and the partition are rigid and no-slip. The working fluid is initially at quiescent. At t = 0, the temperature of the fluid on the left side of the partition is 290K, and that on the right side of the partition is 300K. The detailed initial and boundary conditions for the cases considered here are shown in the Table 1.

3. NUMERICAL SCHEME AND GRID AND TIME STEP DEPENDENCE TESTS

Equations (1) - (4) are solved along with the initial and boundary conditions using the SIMPLE scheme with the help of CFD software FLUENT 6.3. The Finite Volume method has been chosen to discretize the governing equations, with the QUICK scheme (see Leonard and Mokhtari [22]) approximating the advection term. The diffusion terms are discretized using central-differencing with second order accuracy. A second order implicit time-marching scheme has also been used for the unsteady term.

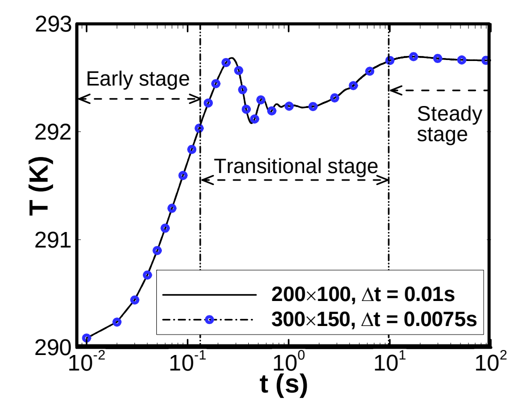


Fig 2. Time series of the temperatures at the point (-0.002m, 0.07m) in the middle thermal boundary layer on the left side of the partition calculated using different meshes and time steps.

Two non-uniform grid sizes, 200×100 and 300×150 with coarser grids in the core and finer grids concentrated in the proximity of all wall and partition boundaries were constructed for grid dependence tests. Fig. 2 plots time series of the temperatures at the point (-0.002m, 0.07 m) in the middle of the thermal boundary layer on the left side of the partition calculated using the two grid systems. Clearly, two solutions almost overlapped. This means that either grid system is able to resolve the transient natural convection in the partitioned cavity and characterize the details of the boundary layers adjacent to the partition and other walls for the present Rayleigh number. With a view to saving computing time, the grid system of 200×100 is adopted in this study.

Two time steps of 0.01s and 0.0075s are tested to examine the effect of the time step. The numerical results obtained using the two time steps are also plotted in Fig. 2. Evidently, the development of the flow is not sensitive to the two tested time steps, with either choice being satisfactory. Accordingly, the larger time step of 0.01*s* is considered to be sufficiently small to capture the transient features of the flow development and is adopted here.

4. RESULTS AND DISCUSSIONS

The transient features of the natural convection flow including the developments of the coupled (cold and hot) thermal boundary layers and intrusions are characterized in the following sections. Furthermore, the heat transfer through the partition and the inclined walls are compared and examined for the effect of the partition on heat transfer for three cases. One case is an isosceles triangular enclosure without a partition in which the boundary conditions on the top and bottom are identical to those for the partitioned case. The other two cases are the right angled triangles of right face and left face which are the half of the domain of isosceles triangle. For the right face right angled triangle the inclined wall is heated and the vertical wall is cooled where the bottom surface is adiabatic. On the other hand, for the left face right angled triangle the top inclined wall is cooled, the vertical wall is heated and the bottom surface is adiabatic. The impact of the partition on the flow and heat transfer in an enclosure can be quantified by comparing the results for partitioned and non-partitioned triangular enclosure. The two right angled triangular enclosures have been considered for the simplicity of analysis and clarity of results. All initial and boundary conditions of temperature are listed in Table 1.

4.1 Transient Flow Development

The time series of the temperature at the point (-0.002*m*, 0.07*m*) has been depicted in Fig. 2. This figure demonstrates the overall development of natural convection from a suddenly generated temperature difference between the fluids on the two sides of the partition to a steady state. As shown in Fig. 2, the flow development can be classified into three main stages: an initial stage, a transitional stage and a steady state stage. At the early stage the conduction dominates the heat transfer and the fluid flow. When the conduction term in the energy equation balances with convection term, the scaling of that time was given by Saha *et al* [17-18] as

$$t_s \sim \frac{(1+\mathrm{Pr})(1+A^2)^{1/2}}{A(Ra\,\mathrm{Pr})^{1/2}}\left(\frac{h^2}{\kappa}\right) \tag{8}$$

It is noted that the temperature growth does not cease immediately after t_s. time for convection to balance conduction. The flow undergoes several overshoots and undershoots during the transitional stage. A second group of waves may also be seen. From the boundary layer, the thermal flows of both sides of the partition discharge fluid into the core of the enclosure. At the end, the fluid inside the enclosure becomes steady state with time.

The time evolution of two isotherms (T =291K and 299K) on both sides of the partition has been shown in Fig. 3. It is seen that the isotherms are shifting horizontally which means the thickness of the coupled thermal boundary layers also increases with time. Saha et al [17-18] and others have pointed out that the thickness of the thermal boundary layer adjacent to an isothermal wall (either vertical or inclined) grows with time according to the scaling relation $\delta_T \sim t^{1/2}\kappa^{1/2}$. Now, on the right side of the partition the cold fluid from the boundary layer has no choice but to travel down along the right horizontal intrusion. Eventually the fluid from the intrusion layer discharges into the core of the enclosure. On the other hand the hot fluid of the boundary layer of the left side of the partition moves up to the top tip and starts travelling along the left inclined wall. This fluid from the boundary layer also discharges into the core of the enclosure.

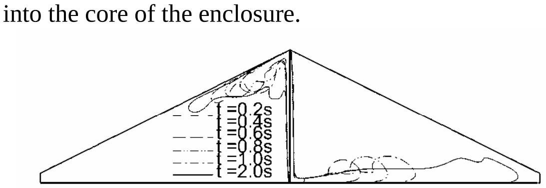


Fig 3**.** Growth of the thermal boundary layers shown as isotherms at different times.

4.2 Development Of Coupled Thermal Boundary Layers

Further transient features of the coupled thermal boundary layers are seen in Fig. 4 which shows comparisons between the time series of the temperatures at the specified point in the middle thermal boundary layers for the partitioned case and the vertical thermal boundary layers for the right angled triangular cases. It is noted that for the purpose of comparing different thermal boundary layers for all the cases, the temperature growths in the thermal boundary layers with reference to the initial interior fluid temperatures are plotted in this figure. Clearly, the temperature growth in the thermal boundary layer adjacent to the partition is consistent with that adjacent to the isothermal vertical wall for the right angled triangular enclosure case except for some variations after the first overshoot (Fig. 4a) or undershoot (Fig. 4b). This is because the initial temperature difference between the partition and the fluid is the same as that between the vertical wall and the fluid for the right angled triangular enclosure case and in the initial stage, the partition is approximately isothermal. This is also confirmed by the following numerical results.

Fig. 5 shows the development of the temperature structure within the coupled thermal boundary layers. It is seen from Fig. 5 that the vertical temperature profiles on the partition change from an initially isothermal to an approximately linear vertical distribution between y = 0.02 m and y = 0.12 m at the steady stage (e.g. t = 50s). In the initial stage there is a temperature difference between the partition and the fluid on the left side of the partition similar to that between the vertical wall and the fluid for the right angled triangular enclosure case. The temperature growth in the thermal boundary layer adjacent to the partition in the initial stage is also similar to that adjacent to the vertical wall in the right angled triangular enclosure case (see Fig. 4). As the bottom surface is adiabatic the temperature of its neighbouring cells responses slowly with time. Because the top end of the partition is in the middle position of the two inclined walls of different temperatures, all temperature contours of different times meet at the initial temperature of the partition on its top.

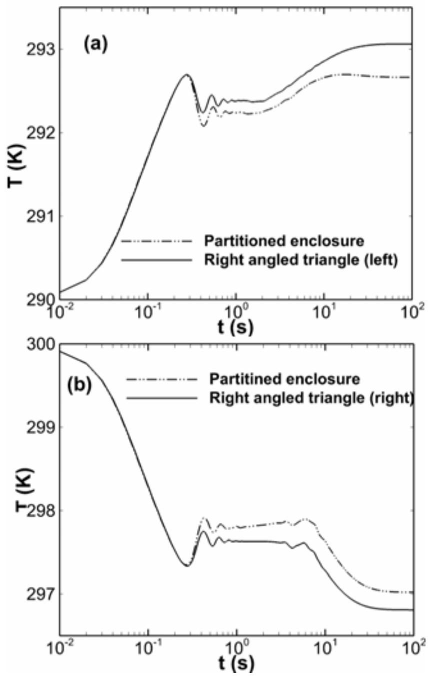


Fig 4. Comparison of the temperature in the partitioned enclosure and the right angled triangular enclosure.

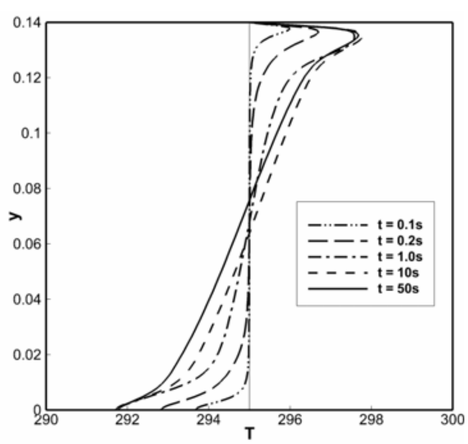


Fig 5. Vertical temperature profiles in the coupled thermal boundary layers at different times.

4.3 Development Of Coupled Thermal Boundary Layers

Even though the partition placed in the geometric symmetric line, the flow may not be symmetrical on two sides of the partition. Because the top walls are inclined and isothermal whereas the bottom surface is horizontal and adiabatic. This asymmetrical flow behaviour can be seen in Fig. 6 and 7. Fig. 6(a) and (b) show the temperature contours and streamlines respectively at t = 1s. The isotherms on the left side of the partition move up to the top inclined wall and have no choice but to travel along the cold inclined wall. On the other hand the isotherms on the right side of the partition travel down along the horizontal adiabatic bottom surface.

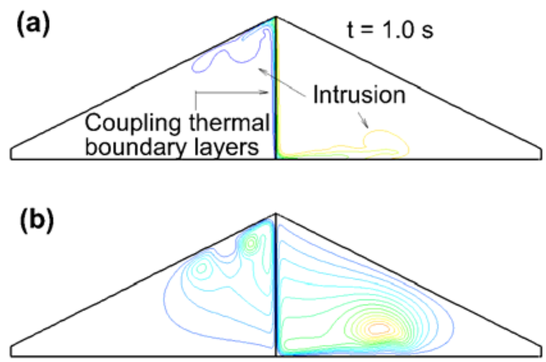


Fig 6. Intrusion in the partitioned cavity. (a) Isotherms (b) Streamlines on both sides of the partition at t = 1.0s, where A = 0.4, Ra = 10^8 and Pr = 0.72.

The asymmetric flow behaviour can also be seen in Fig.7. The top solid line is the temperature time series which has been recorded at (0.002m, 0.07m) and the bottom line is recorded at (-0.002m, 0.07m). Initially the flow seems symmetric (up to approximately 1s), however it shows asymmetric behaviour afterwards.

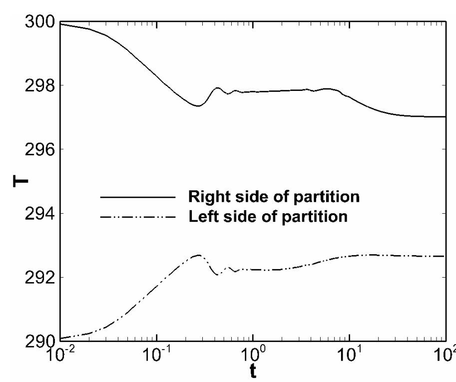


Fig 7. Time series of the temperatures at two centrosymmetric points.

4.4 Heat Transfer

From the literature review it is found that for a laminar flow regime, a vertical partition in a differentially heated cavity apparently depresses heat transfer through the cavity [15]. In this section, the effect of the partition on transient heat transfer are examined. Both the local and the overall Nusselt numbers are calculated on both partition and the inclined walls for the partition enclosure and inclined walls of the non-partitioned isosceles triangular enclosure. Heat transfer is also calculated of the inclined and vertical surfaces of the right angled triangular enclosures. We know from the definition that the Nusselt number represents the ratio of convective heat transfer rate over conductive heat transfer rate. For the present investigation, heat is ultimately transferred from one inclined wall to the other and also from one zone to another through the partition for the partition geometry. The local and the overall Nusselt numbers have been defined as follows:

$$Nu_L = \frac{1}{A}\frac{\partial T}{\partial n} \text{ and } Nu = \frac{1}{A}\int_0^{0.14}\frac{\partial T}{\partial n}dy \tag{9}$$

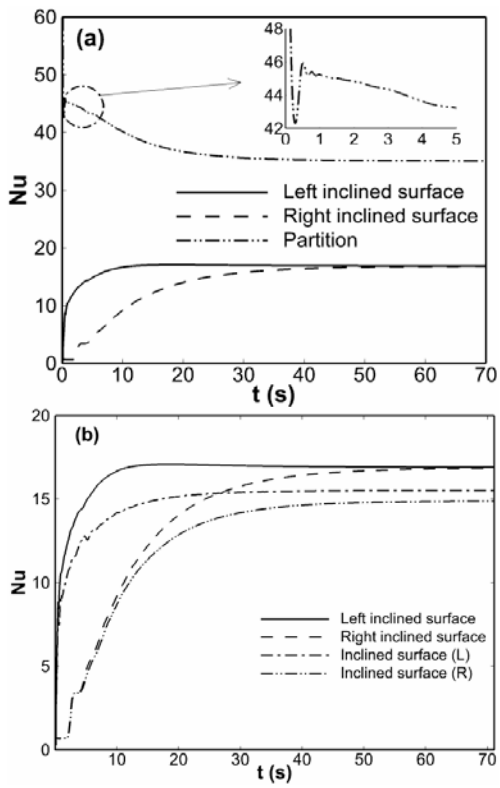


Fig 8. Time series of Nusselt number calculated on both inclined surfaces and on the partition for $Ra = 10^8$.

The effect of the partition on heat transfer through the triangular enclosure has been quantified and is shown in Fig 8. Fig. 8(a) shows the time series of the overall Nusselt numbers calculated on the partition and inclined walls for the partitioned enclosure and Fig. 8(b) shows the Nusselt number for both inclined surfaces for the partitioned enclosure and the inclined surfaces of the right angled triangular enclosures. For the partitioned case, even though the temperature difference between the fluid and isothermal inclined wall at the initial stage is zero, the temperature difference near the top tip increases significantly initially and this influences the overall heat transfer on the surfaces. Initially the heat transfer through the inclined walls of the partitioned and right angled triangular enclosures is the same. However, as time increases the calculated overall Nusselt number on the inclined surface of the partitioned enclosure increases more than that for the right angled triangular enclosure. This is expected, because the vertical wall of the right triangular enclosure is isothermal whereas the partition is diathermal. Moreover, the total temperature difference of two enclosures is not the same. At the steady state stage the Nusselt numbers on two inclined walls of the partitioned enclosure becomes identical because of the mass conservation law.

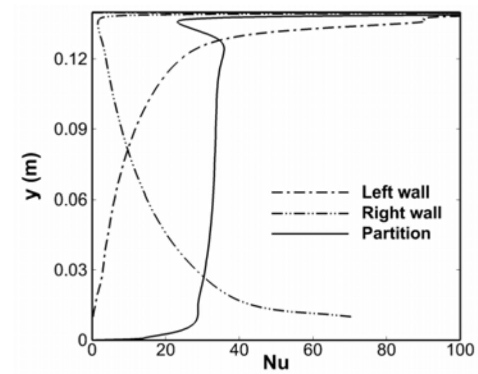


Fig 9. Profiles of the local Nusselt numbers on the partition and the vertical walls of the left and right triangle case.

Fig. 9 shows the profiles of the local Nusselt numbers at the steady state on the partition and two inclined walls of the partitioned triangular enclosure case. It is clear that the local Nusselt number on the inclined walls is non-uniform and in the upstream section of the walls, it is much larger than that in the downstream section due to the formation of the stratification in the core of the enclosure. However, the local Nusselt number on the partition is approximately constant but not so at two ends of the partition. The heat transfer through the partition does not vary with the height except for in the closeness to the top tip and the bottom wall. That means the partition is approximately isoflux at the steady state even though the inclined walls are isotherm which is an interesting phenomena at the steady state condition of the flow.

Time series of the average Nusselt number which are calculated on the inclined walls of the partitioned and non-partitioned triangular enclosure have been presented in Fig. 10. It is noticed that the Nusselt number on the left inclined cold wall (Fig. 10a) of the non-partitioned enclosure is higher at the transitional stage. However, the opposite scenario can be seen in the right heated wall (Fig. 10b). Fig 10(c) shows the difference of the Nusselt number between two identical partitioned and non-partitioned enclosures.

It is interesting to note that for the present Rayleigh number the average heat transfer through the left inclined wall is reduced by 64.44% and through the right wall by 61.67% if a single partition is placed along the middle vertical line of the isosceles triangular enclosure. This calculation is based at the time when the flow is steady state. The rate is much higher than this during the initial stage and the transitional stage of the flow development.

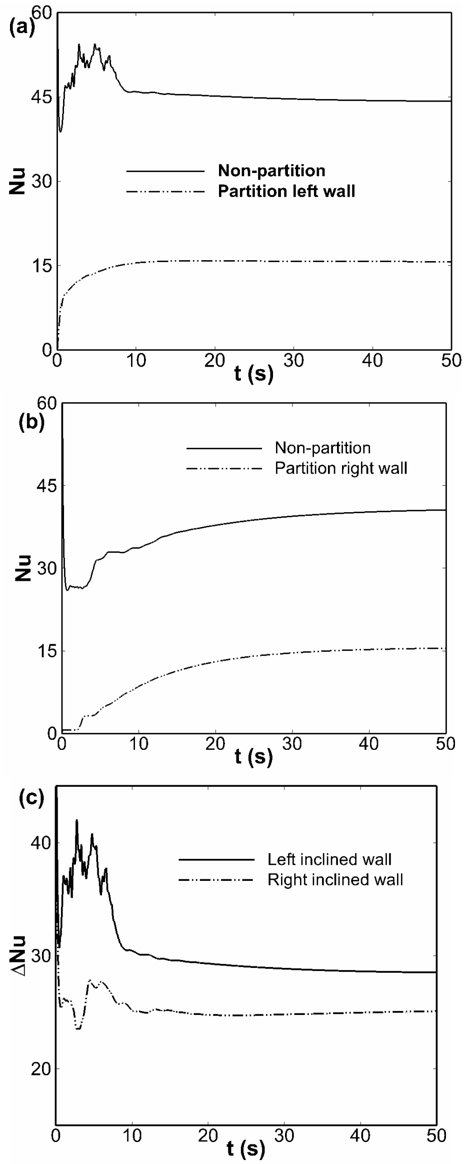


Fig 10. Time series of Nusselt number calculated on (a) left inclined walls , (b) right inclined walls of the partitioned and non-partitioned enclosure and (c) the difference of Nusselt number between partitioned and non-partitioned enclosure.

5 CONCLUSIONS

The coupled thermal boundary layers between the fluids of two zones in an isosceles triangular enclosure separated by a partition induced by suddenly generated temperature difference between the fluid zones are investigated numerically. It is revealed that the development of the transient boundary layers adjacent to the partition are classified into three distinct stages; an initial stage, a transitional stage and a steady state stage. As soon as the fluids of two different temperatures reach the diathermal partition the coupled thermal boundary layers near both sides of the partition start to grow. When the conduction and convection terms in the energy equation are balanced the flow enters into the transitional stage. The thermal fluids discharged from the downstream ends of the coupled thermal boundary layers continuously fill each half of the partitioned cavity from transition stage to the steady state stage. As time progress another thermal boundary layer forms adjacent to the inclined wall. On a close observation of the flow phenomena in the transient process, it may be concluded that the temperature distribution on the partition enclosed by the coupled thermal boundary layers changes from an initially isothermal to an approximately linear profile at the steady state. As a result, an isoflux condition is set up along the partition in the steady state stage.

It is found from the numerical simulations that the heat transfer through the coupled thermal boundary layers is noticeably higher than that through the corresponding thermal boundary layer adjacent to an isothermal inclined wall. Moreover, it was found that for the present Rayleigh number the average heat transfer through the left inclined wall is reduced by approximately 64.44% and that through the right wall by approximately 61.67% if a single partition is considered along the middle vertical line of the triangular enclosure at the steady state stage. The rate is found to be much higher than this during the initial and the transitional stage of the flow development. Multiple partitions are expected to reduce the rate further and this will be studied next. The optimisation of the heat transfer through the walls will be carried out in our future studies. It is expected that these results will help the builder to reconsider and refine their insulation process of the attic shaped building.

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